UNIFACS – Cálculo 4, 2010.2
Exercícios (extras) de Transformada de Laplace
Prof. Adriano Cattai – www.cattai.mat.br/unifacs

1) Determine a transformada de Laplace das seguintes funções:

a) $x^3 + 3\cos\ 2x$ — **R**: $\dfrac{6}{s^4} + \dfrac{3s}{s^2+4}$

b) $5e^{2x} + 7e^{-x}$ — **R**: $\dfrac{5}{s-2} + \dfrac{7}{s+1}$

c) $2x^2 \cos\ x$ — **R**: $\dfrac{4s(s^2-3)}{(s^2+1)^3}$

d) $2x^2 e^{-x} \cos\ x$ — **R**: $\dfrac{4(s+1)[(s+1)^2-3]}{[(s+1)^2+1]^3}$

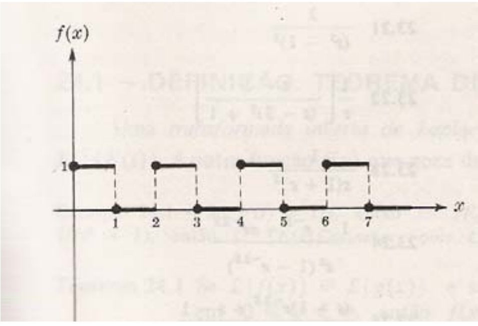
e) $x^2 sen\ 4x$ — **R**: $\dfrac{8(3s^2-16)}{(s^2+16)^3}$

f) $\sqrt{x}e^{2x}$ — **R**: $\dfrac{1}{2}\sqrt{\pi}(s-2)^{-3/2}$

g) $\int_0^x t\ sen\ t\ dt$ — **R**: $\dfrac{2}{(s^2+1)^2}$

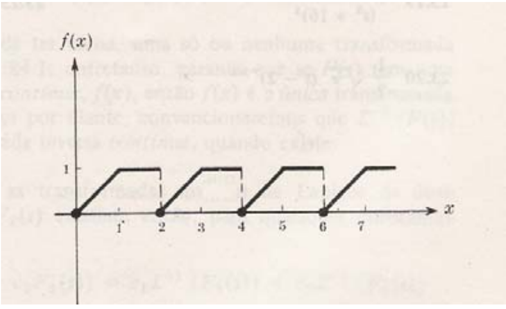
h) $\int_0^x e^{3t} \cos t\, dt$ — **R**: $\dfrac{1}{s}\left[\dfrac{s-3}{(s-3)^2+1}\right]$

i) $f(x)$ na figura seguinte:



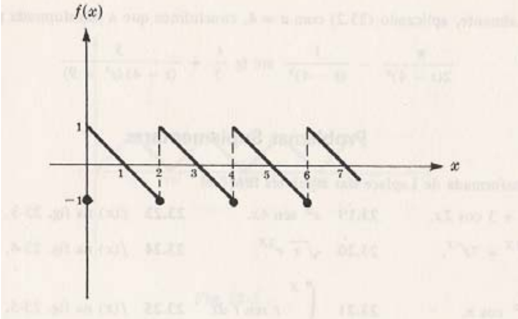
R: $\dfrac{1}{s(1+e^{-s})}$

j) $f(x)$ na figura seguinte:



R: $\dfrac{1-e^{-s}-se^{-2s}}{s^2(1-e^{-2s})}$

l) $f(x)$ na figura seguinte:



R: $\dfrac{(s+1)e^{-2s}+s-1}{s^2(1-e^{-2s})}$

2) Utilizando a tábua das transformadas de Laplace, determine a transformada inversa de:

a) $\dfrac{1}{s+2}$ **R**: e^{-2x} b) $\dfrac{1}{s^2+4}$ **R**: $\dfrac{1}{2} sen\ 2x$

c) $\dfrac{2}{(s-2)^2+9}$ **R**: $\dfrac{2}{3}e^{2x}\cdot sen\ 3x$ d) $\dfrac{s}{(s+1)^2+5}$ **R**: $e^{-x}\cdot\cos\sqrt{5}\,x-\dfrac{1}{\sqrt{5}}\cdot e^{-x}\cdot sen\ \sqrt{5}\,x$

e) $\dfrac{2s+1}{(s-1)^2+7}$ **R**: $2e^{x}\cdot\cos\sqrt{7}\,x+\dfrac{3}{\sqrt{7}}\cdot e^{x}\cdot sen\ \sqrt{7}\,x$

f) $\dfrac{1}{2s^2+1}$ **R**: $\dfrac{1}{\sqrt{2}} sen\ \dfrac{1}{\sqrt{2}}x$

3) Completando o quadrado, determine a transformada inversa de Laplace de:

a) $\dfrac{1}{s^2-2s+2}$ **R**: $e^{x}\cdot sen\,x$ b) $\dfrac{s+3}{s^2+2s+5}$ **R**: $e^{-x}\cdot\cos\ 2x+e^{-x}\cdot sen 2x$

c) $\dfrac{s}{s^2-s+17/4}$ **R**: $e^{(1/2)x}\cdot\cos\ 2x+\dfrac{1}{4}\cdot e^{(1/2)x}\cdot sen\ 2x$

d) $\dfrac{s+1}{s^2+3s+5}$ **R**: $e^{-(3/2)x}\cdot\cos\dfrac{\sqrt{11}}{2}x-\dfrac{1}{\sqrt{11}}\cdot e^{-(3/2)x}\cdot sen\ \dfrac{\sqrt{11}}{2}x$

4) Utilize o método das frações parciais para decompor:

a) $\dfrac{2s^2}{(s-1)\cdot(s^2+1)}$ **R**: $\dfrac{1}{s-1}+\dfrac{s+1}{s^2+1}$ b) $\dfrac{1}{s^2-1}$ **R**: $\dfrac{1/2}{s-1}+\dfrac{-1/2}{s+1}$ c) $\dfrac{2}{(s^2+1)\cdot(s-1)^2}$ **R**: $\dfrac{s}{s^2+1}+\dfrac{-1}{s-1}+\dfrac{1}{(s-1)^2}$

5) Determine as transformadas inversas de Laplace das funções do exercício anterior.

R: a) $e^{x}+\cos\ x+sen\,x$ b) $\dfrac{1}{2}e^{x}-\dfrac{1}{2}e^{-x}$ c) $\cos\ x-e^{x}+x\cdot e^{x}$

6) Determine as transformadas inversas de Laplace de:

a) $\dfrac{2s-13}{s\cdot(s^2-4s+13)}$ **R**: $-1+e^{2x}\cdot\cos\ 3x$ b) $\dfrac{2\cdot(s-1)}{s^2-s+1}$ **R**: $2e^{(1/2)x}\cdot\cos\dfrac{\sqrt{3}}{2}x-\dfrac{2}{\sqrt{3}}\cdot e^{(1/2)x}\cdot sen\ \dfrac{\sqrt{3}}{2}x$

c) $\dfrac{s}{(s^2+9)^2}$ **R**: $\dfrac{1}{6}x\cdot sen 3x$ (ver Tábua, item 12) d) $\dfrac{1}{2\cdot(s-1)\cdot(s^2-s-1)}=\dfrac{1/2}{(s-1)\cdot(s^2-s-1)}$

R: $-\dfrac{1}{2}e^{x}+\dfrac{1}{2}e^{(1/2)x}\cosh\dfrac{\sqrt{5}}{2}x+\dfrac{1}{2\sqrt{5}}\cdot e^{(1/2)x}\cdot senh\ \dfrac{\sqrt{5}}{2}x$ e) $\dfrac{s}{2s^2+4s+5/2}=\dfrac{(1/2)\cdot s}{s^2+2s+5/4}$

R: $\dfrac{1}{2}e^{-x}\cdot\cos\ \dfrac{1}{2}x-e^{-x}\cdot sen\ \dfrac{1}{2}x$

7) Utilize transformadas de Laplace para resolver os seguintes problemas de valor inicial:

a) $y'+2y=0; y(0)=1$ **Resposta**: $y=e^{-2x}$
b) $y'+2y=2; y(0)=1$ **Resposta**: $y=1$
c) $y'+2y=e^{x}; y(0)=1$ **Resposta**: $y=\dfrac{2}{3}e^{-2x}+\dfrac{1}{3}e^{x}$
d) $y'+2y=0; y(1)=1$ **Resposta**: $y=e^{-2(x-1)}$
e) $y'+5y=0; y(1)=0$ **Resposta**: $y=0$

f) $y''-y=0; y(0)=1, y'(0)=1$ **Resposta**: $y=e^{x}$

g) $y''-y=sen\ x; y(0)=0, y'(0)=1$ **Resposta**: $y=\frac{3}{4}e^{x}-\frac{3}{4}e^{-x}-\frac{1}{2}sen\,x$

h) $y''-y=e^{x}; y(0)=1, y'(0)=0$ **Resposta**: $y=\frac{1}{4}e^{x}+\frac{3}{4}e^{-x}+\frac{1}{2}xe^{x}$

i) $y''+2y'-3y=sen\ 2x; y(0)=0, y'(0)=0$ **Resposta**: $y=\frac{1}{10}e^{x}-\frac{1}{26}e^{-3x}-\frac{4}{65}\cos\ 2x-\frac{7}{65}sen\ 2x$

j) $y''+y=sen\,x; y(0)=0, y'(0)=2$ **Resposta**: $\frac{5}{2}sen\,x-\frac{1}{2}x\cos x$

k) $y''+y'+y=0; y(0)=4, y'(0)=-3$ **Resposta**: $y=4e^{-(1/2)x}\cos\frac{\sqrt{3}}{2}x-\frac{2}{\sqrt{3}}e^{-(1/2)x}sen\ \frac{\sqrt{3}}{2}x$

l) $y''+2y'+5y=3e^{-2x}; y(0)=1, y'(0)=1$ **Resposta**: $y=\frac{3}{5}e^{-2x}+\frac{2}{5}e^{-x}\cos\ 2x+\frac{13}{10}e^{-x}\ sen\,2x$

m) $y''+5y'-3y=u(x-4); y(0)=0, y'(0)=0$

Resposta: $y=\left[-\frac{1}{3}+\frac{1}{3}e^{-(5/2)\cdot(x-4)}\cosh\frac{\sqrt{37}}{2}(x-4)+\frac{5}{3\sqrt{37}}e^{-(5/2)\cdot(x-4)}senh\ \frac{\sqrt{37}}{2}(x-4)\right]\cdot u(x-4)$

n) $y''+y=0; y(\pi)=0, y'(\pi)=-1$ **Resposta**: $y=sen\ x$

o) $y'''-y=5; y(0)=0, y'(0)=0, y''(0)=0$ **Resposta**: $y=-5+\frac{5}{3}e^{x}+\frac{10}{3}e^{-(1/2)x}\cos\frac{\sqrt{3}}{2}x$

p) $y^{(4)}-y=0; y(0)=1, y'(0)=0, y''(0)=0, y'''(0)=0$ **Resposta**: $y=\frac{1}{4}e^{x}+\frac{1}{4}e^{-x}+\frac{1}{2}\cos\ x$