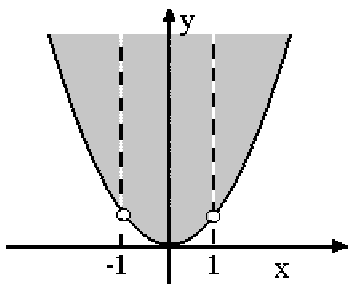
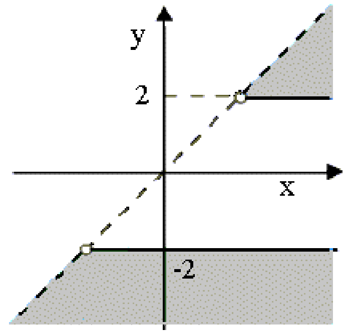
Respostas

1)

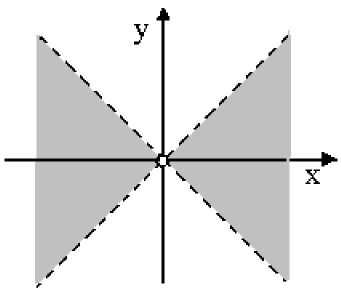
a) $\{(x,y) \in R^2;\ x^2 - 1 \neq 0 \text{ e } y \geq x^2\}$



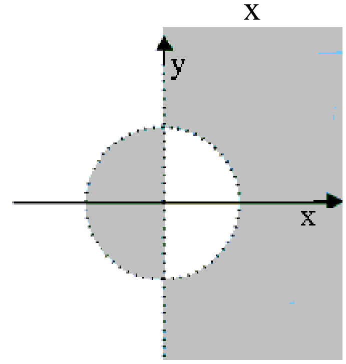
b) $\{(x,y) \in R^2;\ y \geq 4 \text{ e } x > y\ \}$



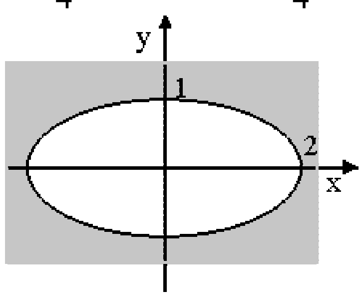
c) $\{(x,y) \in R^2;\ x^2 - y^2 > 0\}$



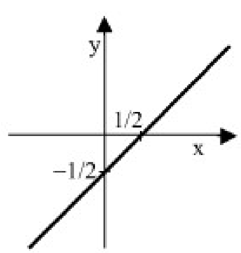
d) $\{(x,y) \in R^2;\ x \neq 0 \text{ e } \dfrac{x^2 + y^2 - 1}{x} > 0\,\}$



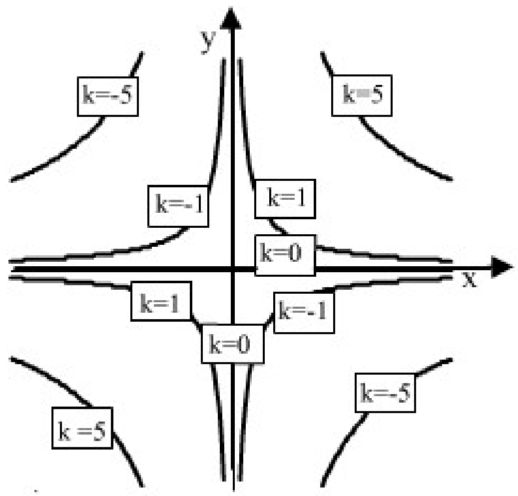
e) $\{(x,y) \in R^2; \dfrac{x^2}{4} + y^2 \leq -1 \text{ ou } \dfrac{x^2}{4} + y^2 \geq 1\,\}$



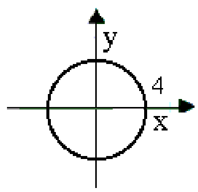
2) A reta $x - y = 1/2$



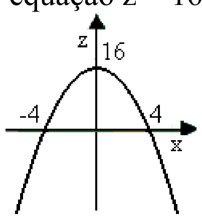
3) Seja $z = k$.
$k = 0$: As retas $x = 0$ e $y = 0$.
$k = 1, 5, -1, -5$: Temos respectivamente as curvas
$y = 1/x$; $y = 5/x$; $y = -1/x$;
$y = -5/x$



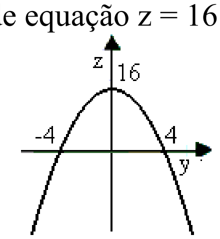
4) a)
$D(f) = R^2$
$G(f) \cap xOy$:
círculo de equação $x^2 + y^2 = 16$



$G(f) \cap xOz$:
Parábola de equação $z = 16 - x^2$



$G(f) \cap yOz$:
Parábola de equação $z = 16 - y^2$



Curvas de nível
Para $z = k$,
$k < 16$: círculo de equação $x^2 + y^2 = \left(\sqrt{16-k}\right)^2$
$k = 16$: ponto $(0,0)$
$k > 16$: $\varnothing$

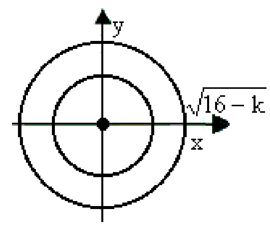
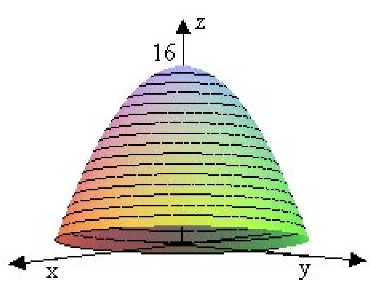


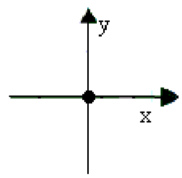
Gráfico: (um parabolóide de revolução)



4) b)

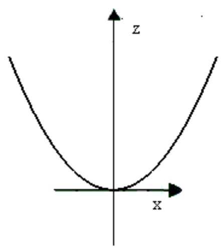
$D(f) = R^2$

$G(f) \cap xOy$: ponto (0,0)



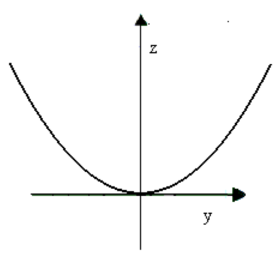
$G(f) \cap xOz$:

Parábola de equação $z = 9x^2$



$G(f) \cap yOz$:

Parábola de equação $z = 4y^2$



Curvas de nível

Para $z = k$,

$k > 0$: elipse de equação $\dfrac{x^2}{(\sqrt{k}/3)^2} + \dfrac{y^2}{(\sqrt{k}/2)^2} = 1$

$k = 0$: ponto (0,0)

$k < 0$: $\varnothing$

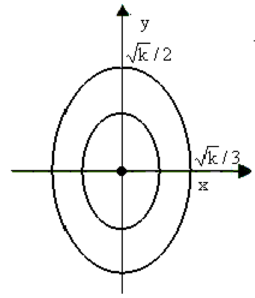
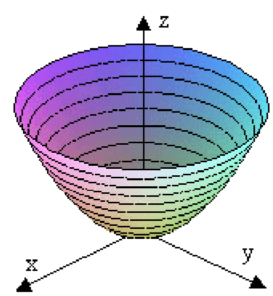


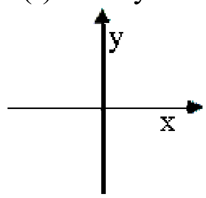
Gráfico: (Um parabolóide elíptico)



4) c)

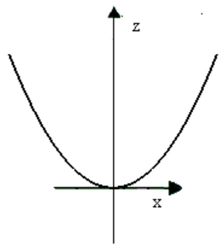
$D(f) = R^2$

$G(f) \cap xOy$: o eixo Oy

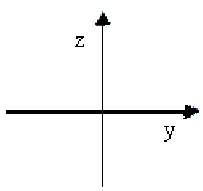


$G(f) \cap xOz$:

Parábola de equação $z = x^2$



$G(f) \cap yOz$: o eixo Oy



Curvas de nível

Para $z = k$,

$k > 0$: as retas $x = \sqrt{k}$ e $x = -\sqrt{k}$

$k = 0$: o eixo OY

$k < 0$: $\varnothing$

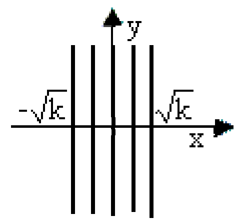
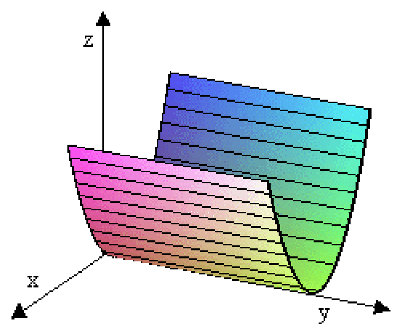


Gráfico: (uma superfície cilíndrica)

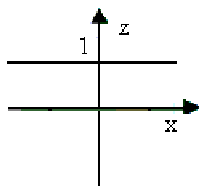


4) d)

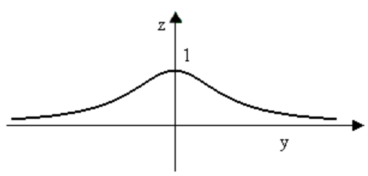
$D(f) = R^2$

$G(f) \cap xOy$: $\varnothing$

$G(f) \cap xOz$: a reta $z = 1$



$G(f) \cap yOz$: a curva $z = 1/(1+y^2)$



Curvas de nível

Para $z = k$,

$0 < k < 1$: as retas $y = \sqrt{1/k - 1}$ e $y = -\sqrt{1/k-1}$

$k = 1$: o eixo Ox

$k > 1$ ou $k \leq 0$: $\varnothing$

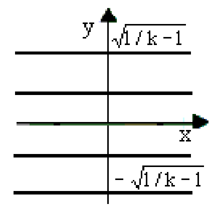
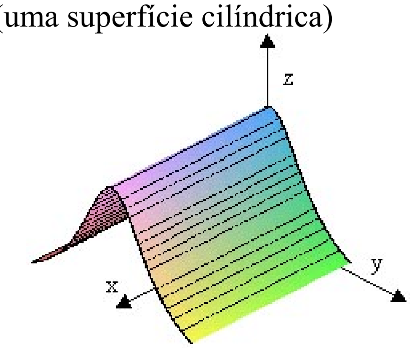
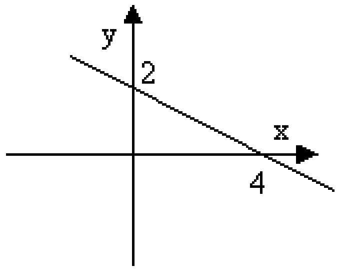


Gráfico: (uma superfície cilíndrica)



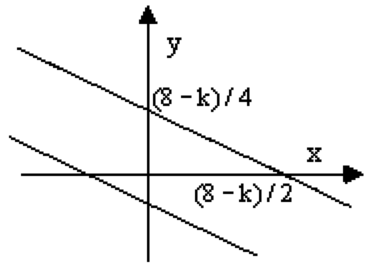
4) e)
$D(f) = R^2$

$G(f) \cap xOy$: a reta $y = -\frac{x}{2} + 2$

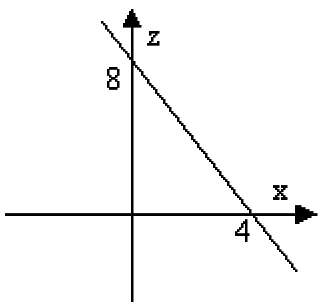


Curvas de nível
Para $z = k$,

$\forall\, k \in R$: a reta $y = -\frac{x}{2} + \frac{8-k}{4}$



$G(f) \cap xOz$: a reta $z = -2x + 8$



$G(f) \cap yOz$: a reta $z = -4y + 8$

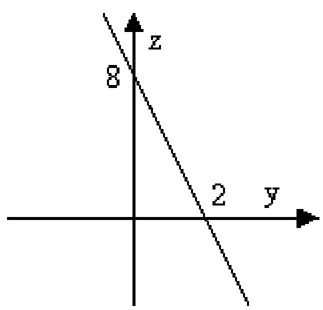
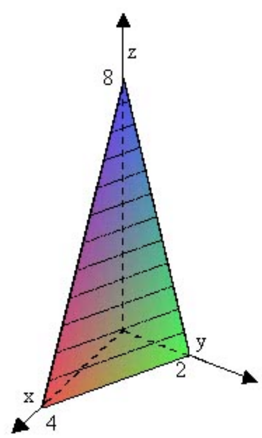


Gráfico: (um plano)

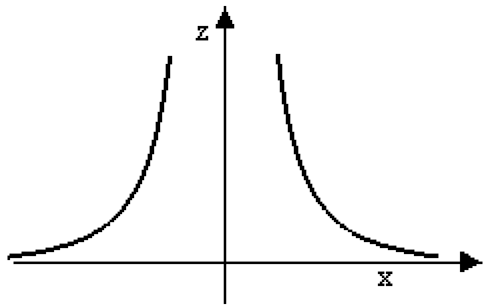


4) f)

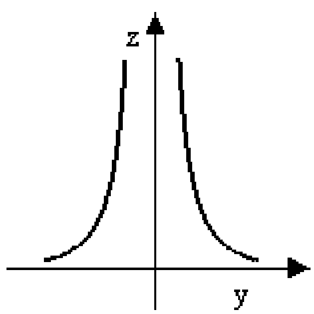
$D(f) = R^2 - \{(0,0)\}$

$G(f) \cap xOy$: $\varnothing$

$G(f) \cap xOz$: a curva $z = 4/x^2$



$G(f) \cap yoz$: a curva $z = 1/y^2$



Curvas de nível

Para $z = k$,

$k > 0$: elipse de equação $\dfrac{x^2}{(2/\sqrt{k})^2} + \dfrac{y^2}{(1/\sqrt{k})^2} = 1$

$k \leq 0$: $\varnothing$

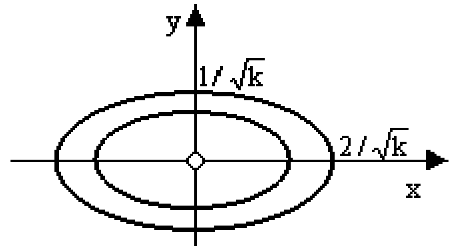
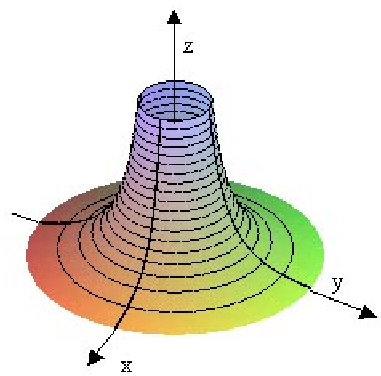


Gráfico:

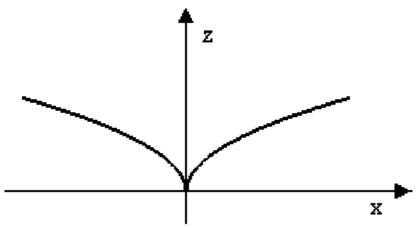


4) g) $f(x,y)=\sqrt[4]{x^2+y^2}$

$D(f) = R^2$

$G(f) \cap xOy$: o ponto (0,0)

$G(f) \cap xOz$: a curva $z = \sqrt{|x|}$



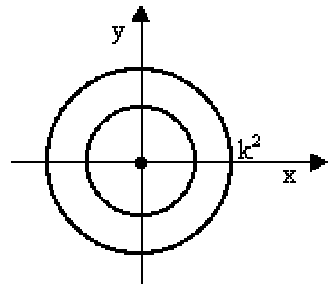
Curvas de nível

Para z = k,

k > 0: circulo de equação $x^2 + y^2 = (k^2)^2$

k = 0: o ponto (0,0)

k < 0: $\varnothing$



$G(f) \cap yOz$: : a curva $z = \sqrt{|y|}$

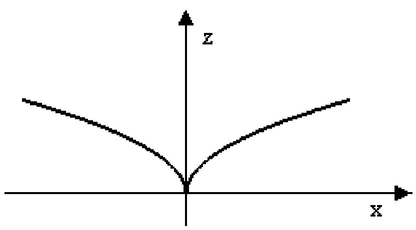
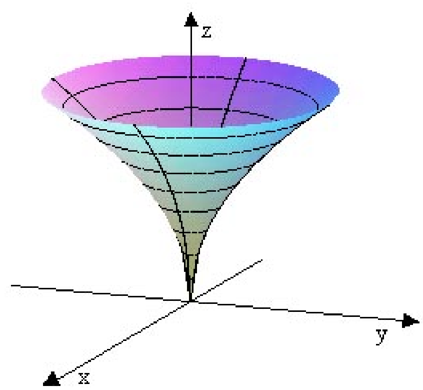


Gráfico: (uma superfície de revolução)



5) a) zero b) 4 c) zero d) zero

7) a) R^2 - { (-1,0) } b) R^2

8) **a**) $f_x(P_o) = e(1+\ln 2)$, $f_y(P_o) = \dfrac{e}{2}$ b) $f_x(P_o) = -1$, $f_y(P_o) = 0$

c) $f_x(P_o) = \dfrac{\sqrt{3}}{3}$, $f_y(P_o) = \dfrac{-\sqrt{3}}{12}$ d) $f_x(P_o) = \dfrac{1}{4}, f_y(P_o) = 1, f_z(P_o) = 1$

e) $f_x(P_o) = 0, f_y(P_o) = 5$, $\nexists\ f_x(P_1)$ e $\nexists\ f_y(P_1)$. f) $f_x(0,0) = 0$ e $f_y(0,0) = 0$.

10) a) 2 b) 2/3 **11**) 17 **12**) a) $-2xe^{-x^4}$ b) $4e^{-4}$

13) a) $\begin{cases} x = 1 \\ y = ez \end{cases}$ b) $\begin{cases} y = 2 \\ 4x + ez = 6 \end{cases}$

14) a) 200 b) 400 **15**) a) $\frac{21\pi}{\sqrt{58}}$ b) $\frac{67\pi}{\sqrt{58}}$ **16**) a) 19000 b) 1;2 c) Manter B constante

18) $\frac{\partial^2 z}{\partial y \partial x} = \frac{e^x}{y} - \cos(x)\operatorname{sen}(y) = \frac{\partial^2 z}{\partial x \partial y}$

19) a) $\frac{\partial^2 f}{\partial y \partial x}(2,3) = 2$ b) i) $\frac{\partial f}{\partial u}(0,1) = \frac{17}{6}$ ii) $\frac{\partial y}{\partial v}(0,1) = -54$

21) a) $df(x,y) = \frac{dx}{\sqrt{y^2 - x^2}} - \frac{x\,dy}{y\sqrt{y^2 - x^2}}$

b) $df(x,y,z) = (e^{yz} + yze^{xz})dx + (xze^{yz} + e^{xz})dy + (xye^{yz} + xye^{xz})dz$

22) a) Plano tangente**:** $z = -\frac{1}{2}(x-1) + (y-1) + \frac{\pi}{4}$; reta normal: $\begin{cases} x = 1 - \frac{1}{2}t \\ y = 1 + t \\ z = \frac{\pi}{4} - t \end{cases}$

b) Plano tangente**:** $z = \frac{x}{e}$; reta normal: $\begin{cases} x = e - e^{-1}t \\ y = t \\ z = 1 - t \end{cases}$

23) $z = (x - 3) + 3(y - 1) + 3$

24) $\beta = 2$

25) a) $\frac{\partial g}{\partial x}(2,1) = 2$**;** $\frac{\partial g}{\partial y}(2,1) = -6$ b) $\frac{\partial h}{\partial z}(1,1,2) = 8$

26) a) -2/5 b) zero c) -6/169

27) a) $\frac{3\sqrt{34}}{34}\vec{i} - \frac{5\sqrt{34}}{34}\vec{j}$; $\sqrt{306}$ **b)** $\frac{2}{\sqrt{4+\pi^2}}\vec{i} - \frac{\pi}{\sqrt{4+\pi^2}}\vec{j}$; $\sqrt{4+\pi^2}$

28) $-\frac{28}{\sqrt{2}}$; $(-12,-16)$; $\lambda(4\vec{i} - 3\vec{j})$; $\lambda \neq 0$

29) $-\frac{178}{\sqrt{14}}$; $4\vec{i} - 8\vec{j} + 54\vec{k}$, $\sqrt{2996}$

30) $\vec{\nabla} z(2,0) = \left(0, -\frac{1}{\ln 2}\right)$

31) a) $2\vec{i} + 2\vec{j}$; $x + y - 2 = 0$ b) $4\vec{i} + \vec{j}$; $4x + y - 3 = 0$

32) a) $x+2y+3z=6; \dfrac{x-1}{2}=\dfrac{y-1}{4}=\dfrac{z-1}{6}$ b) $6x+3y+2z=18;\ x-1=2y-4=3z-9$

c) $\pi x+18y-\pi z=6\pi;\ \dfrac{x-1}{\pi}=\dfrac{y-\frac{\pi}{6}}{18}=-\dfrac{z+2}{\pi}$ d) $z=8;\ (x,y,z)=(2,2,0)+t(0,0,1);\ t\in IR$

e) $z-x=0;\ (x,y,z)=(1,1,1)+t(1,0,-1);\ t\in IR$

33) $x+4y+6z=21$ e $x+4y+6z=-21$

34) Não existe plano tangente à superfície paralelo ao plano $z = 0$. Os planos tangentes à superfície nos pontos $(1,1,0)$ e $(1,-1,0)$ são paralelos ao plano $y = 0$. Os planos tangentes à superfície nos pontos $(0,0,0)$ e $(2,0,0)$ são paralelos ao plano $x = 0$.

35) a) $(-66, 107, \dfrac{143}{6})$ b) $(6,4,-6)$.

36)

a) 42 b) $\dfrac{1}{2}$ c) $2+\dfrac{\pi^2}{2}$ d) $\dfrac{(e-1)}{2}$

e) $1-\cos(1)$ f) $\dfrac{20-\ln 3}{\ln^2 3}$ g) $\dfrac{1}{3}$ h) $\sqrt{3}-\dfrac{8}{3}$

37) a) $\dfrac{864}{5}$ b) $\dfrac{3\pi}{2}$ c) -2

38) a) $\dfrac{1}{12}$ b) 72 **39)** $\dfrac{45}{8}$

40) a) $\dfrac{128}{3}$ b) $\dfrac{512}{3}$ c) $\dfrac{128}{3}$ d) $\dfrac{abc}{3}$ e) 2 f) $\dfrac{10}{3}$